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A Chapman-Kolmogorov approach for diffusion in an expanding medium

S. B. Yuste[1], E. Abad[2]*, F. Le Vot[1], C. Escudero[3]

[1]Universidad de Extremadura, Badajoz, Spain
[2]Universidad de Extremadura, Mérida, Spain
[3]Universidad Autónoma de Madrid, Madrid, Spain
*eabad@unex.es

A vast majority of studies devoted to diffusion assume that it takes place in a static medium. However, in a number of cases the medium growth or contraction takes place in time scales that are relevant for diffusive transport. For instance, in developmental biology, the formation of biological structures (e.g. the pigmentation of skin) occurs during tissue growth. Another example is the diffusion of cosmic rays in the expanding universe. Typically, two types of approaches are used to derive the relevant diffusion equation, namely a) a continuity equation based on mass conservation arguments and particle fluxes that depend linearly on concentrations b) a coarse-grained master equation formalism. Here, we present an alternative approach based on a random walk model that naturally leads to a Chapman-Kolmogorov equation [1]. For the case of a 1D power-law expansion, we find striking crossover effects depending on the exponent γ characterizing the medium growth. The mean square displacement grows linearly in time when $\gamma<1/2$, but is proportional to $t^{2\gamma}$ when $\gamma>1/2$. In the marginal case $\gamma=1/2$ there is a logarithmic correction to the linear increase in time. Beyond this, there are further consequences at the level of propagator. When $\gamma>1/2$, particles are not able to efficiently spread across the whole medium, and this leads to strong localization effects and poor mixing of two diffusive pulses that have a minimum initial separation (see Fig. 1). In this case, a strong memory of the initial condition persists in the long time regime. Finally, the probability for a diffusive walker starting from the center of a growing sphere to cross its surface becomes less than one when $\gamma>1/2$. When $\gamma=1/2$, the n-th order moment of the first passage time may or may not exist depending on whether the diffusivity D exceeds an n-dependent threshold value. We discuss possible implications of our findings for a number of real systems as well as some preliminary results for anomalous diffusion processes.

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