4. Diffusion and Spreading beyond hard natural science

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Gero Vogl
Fakultät für Physik
Dynamik kondensierter Systeme
Universität Wien, Austria

1. **A few words on history**
2. **Differential equations**
3. **Agent based computer simulations**

Textbooks on spread and diffusion Okubo 1980, Murray 1989

1. History of „interdisciplinary diffusion“

In 1827 Robert Brown, a British botanist, found that pollen immersed in a liquid performed an unceasing (endless) motion.

Brown could exclude that this motion was “vitality” as argued in the beginning.

Brown suspected it was physics and many decades later this “Brownian motion” was the stepping stone for Albert Einstein’s ingenious description of the phenomenon.

In 1855 Adolf Fick, a German physiologist, dissolved salt in water, studied the change of salt distribution in time and wrote down the equations governing that phenomenon called diffusion.

Brown and Fick, two eminent scientists, both neither mathematician nor physicist.
So why should the phenomenon of endless motion be a domain of mathematics or physics?

1827
Robert Brown
botanist
„endless motion“

1855
Adolf Fick

$$\frac{\partial c(x,t)}{\partial t} = D\frac{\partial^2 c(x,t)}{\partial x^2}$$

2. Differential equations

In 1951, J.G.Skellam, a “biometrist”, wrote:

“… apparent that many ecological problems have a physical analogue and that the solution of these problems will require treatment with which we are already very familiar.”

$$\frac{\partial c(x,t)}{\partial t} = D\frac{\partial^2 c(x,t)}{\partial x^2} \qquad \frac{\partial c(r,t)}{\partial t} = \frac{D}{r}\frac{\partial}{\partial r}\left(r\frac{\partial c(r,t)}{\partial r}\right)$$

Mostly one-dimensional

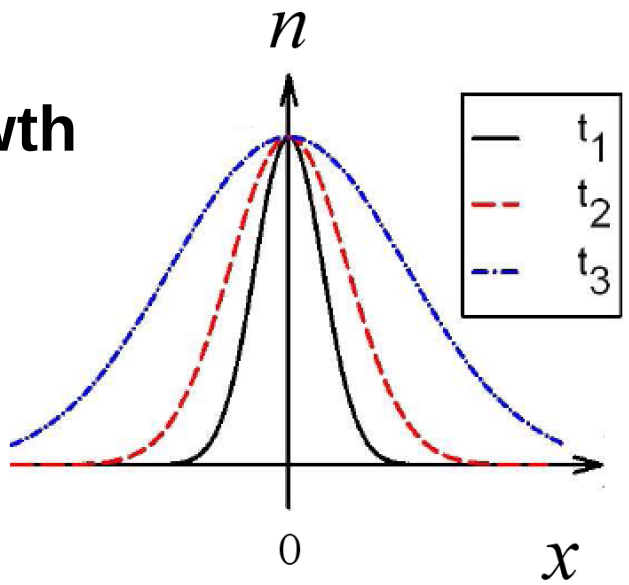
But: “Unlike most of the particles considered by physicists living organisms reproduce”

$$\frac{\partial c(x,t)}{\partial t} = D\frac{\partial^2 c(x,t)}{\partial x^2} + \alpha c(x,t) \qquad \frac{\partial c(r,t)}{\partial t} = \frac{D}{r}\frac{\partial}{\partial r}\left(r\frac{\partial c(r,t)}{\partial r}\right) + \alpha c(r,t)$$

α = growth rate

Malthusian growth

$c(x,t)$= density of people,



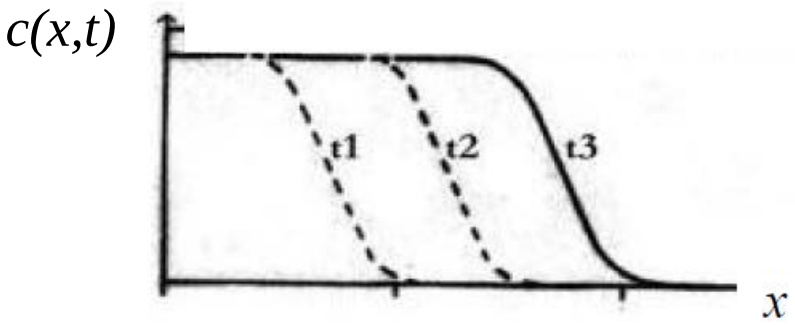
Gauss

No longer Gauss

Logistic growth

K = carrying capacity

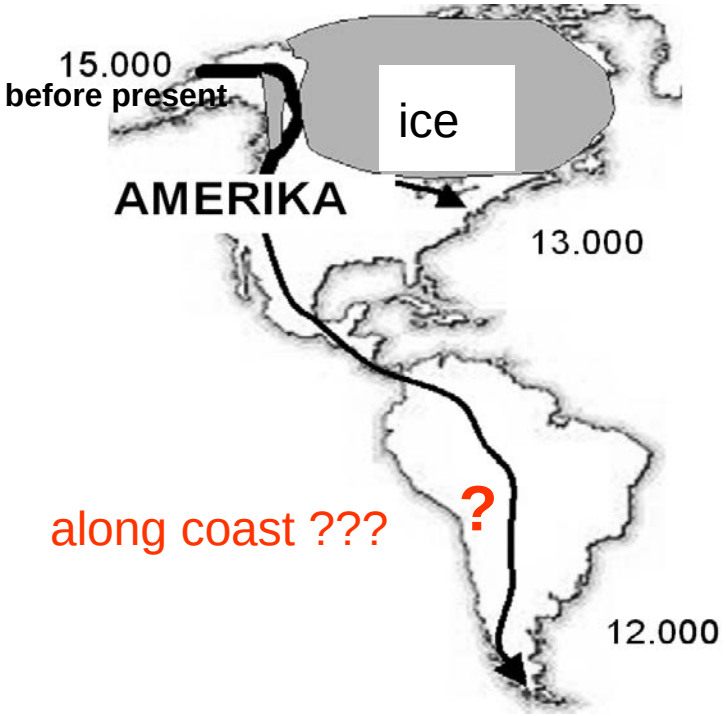
$$\frac{\partial c(x,t)}{\partial t} = D\frac{\partial^2 c(x,t)}{\partial x^2} + \alpha\left(1 - \frac{c(x,t)}{K}\right)c(x,t)$$



$$v = 2\sqrt{D\alpha}$$

Wave of advance

Spread of Paleoindians from Alaska to Patagonia



My own ideas (2007): follow Einstein Equation $\langle x^2(t) \rangle = 2Dt$

→estimate D from mating distance x and length of generation t

→ estimate velocity of wave of advance $v = \sqrt{2D\alpha}$ (growth rate α)

Present state genetics

Willerslev, Copenhague, Science July 21, 2015

Reich et al., Harvard, Nature July 21, 2015

Reaction-diffusion

J.G. Skellam (1951):

"Unlike most of the particles considered by physicists living organisms reproduce and interact."

$$\frac{\partial c_1(x,t)}{\partial t} = D_1 \frac{\partial^2 c_1(x,t)}{\partial x^2} - k_{12} c_1(x,t) + k_{21} c_2(x,t)$$

$$\frac{\partial c_2(x,t)}{\partial t} = D_2 \frac{\partial^2 c_2(x,t)}{\partial x^2} - k_{21} c_2(x,t) + k_{12} c_1(x,t)$$

"As a result the equations of mathematical ecology are often of a new and unusual kind."

Coupled **reaction-diffusion equations**

Please note:

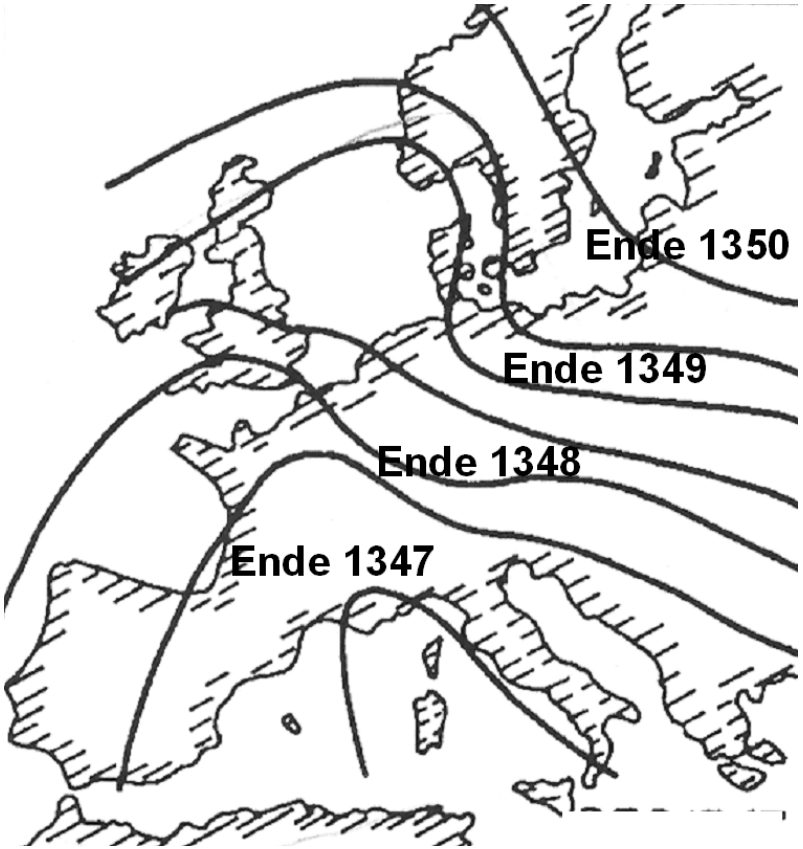
c can be replaced by S, I, f, h, u, p, n…..

and $\partial^2 c / \partial x^2$ by Laplace operator Δ

Epidemies:

W.L.Langer, *Scientific American* (1964)

Black death



J.V.Noble, *Nature* (1974) Geographic and temporal development of plagues

Time dependence of infection

$$\frac{\partial S}{\partial t} = -\alpha IS$$

$$\frac{\partial I}{\partial t} = \alpha IS - \beta I$$

S…population density of susceptible individuals
I…..population density of infected individuals
α….infection rate („cross section“)
β….mortality rate

+ Spread (diffusion) 1-dimensional

$$\frac{\partial S}{\partial t} = -\alpha IS + D\frac{\partial^2 S}{\partial x^2}$$

$$\frac{\partial I}{\partial t} = \alpha IS - \beta I + D\frac{\partial^2 I}{\partial x^2}$$

D…Diffusivity of people

Dirk Brockmann *Epidemics spreading*

Poster 11: V.L. de Rioja, J. Fort and N. Isern
Spread of virus infections

Archaeology

Reaction diffusion equations have been introduced into ethnology/archaeology in the famous work by A.J.Ammerman and L.L.Cavalli-Sforza (1984).

Ammerman and Cavalli-Sorza described the invasion of agriculturalists (“farmers”), the people of the Neolithics between 8.000 and 6.000 years ago, from the Near East by a one-dimensional “wave of advance” from the south-east towards the north-west of Europe. But where it really people that advanced – “demic diffusion”? Or was it the culture of farming which converted the people of the Paleolithics (“hunters”) - “cultural diffusion”?

At that time the authors had no more genetics at their hand than blood groups and archaeological indications for adapting their diffusion equations.

Joaquim Fort
The Neolithic Transition: Diffusion of People or Diffusion of Culture?

Poster 49: F. Silva, C.J. Stevens, A. Weisskopf, C. Castillo, L. Qin, A. Bevan and D.Q. Fuller
Modelling the geographical origin of rice cultivation in Asia using the Rice Archaeological Database

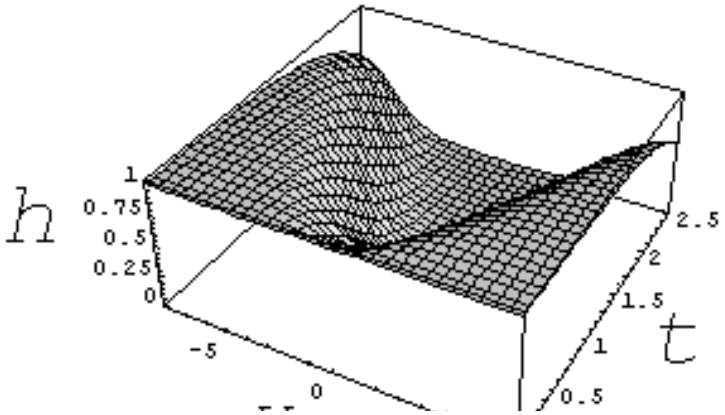
spread growth interaction

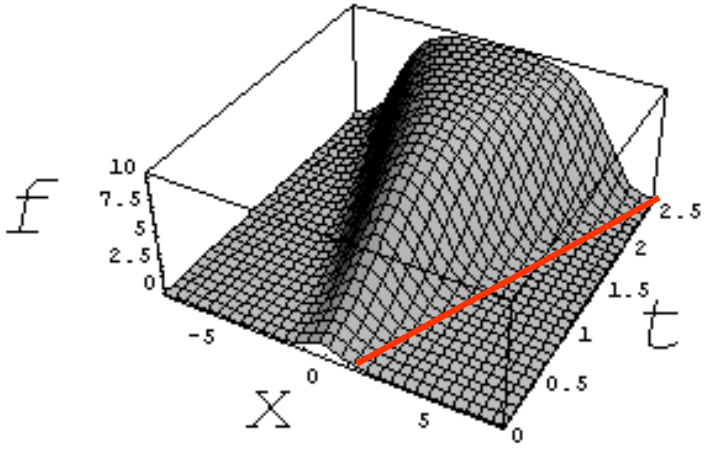
$$\frac{\partial f}{\partial t} = D_f \frac{\partial^2 f}{\partial x^2} + \alpha_f \cdot f\left(1 - \frac{f}{K_f}\right) + \beta \cdot f \cdot h$$

f = density of „farmers“ (neolithic), h = density of „hunters“ (paleolithic), β = conversion coefficient

K=carrying capacity

$$\frac{\partial h}{\partial t} = D_h \frac{\partial^2 h}{\partial x^2} + \alpha_h \cdot h\left(1 - \frac{h}{K_h}\right) - \beta \cdot f \cdot h$$





logistic growth

$$v = 2\sqrt{D\alpha}$$

wave of advance

Language

Anne Kandler

Analysing Language Shift: The Example of Scottish Gaelic

Coupled reaction-diffusion equations with growth and interaction

$$\frac{\partial u_1}{\partial t} = D_1 \Delta u_1 + a_1 \cdot u_1 \left(1 - \frac{u_1}{K}\right) + c_{12} \cdot u_2 \cdot u_1 - \ldots$$

$$\frac{\partial u_2}{\partial t} = D_2 \Delta u_2 + a_2 \cdot u_2 \left(1 - \frac{u_2}{K}\right) + c_{21} \cdot u_2 \cdot u_1 - \ldots$$

Poster 24: Isern and J. Fort *Fronts of language replacement*

3. „Agent based“ computer simulations

Cellular automaton – Monte Carlo

Studying spread by stochastically choosing the action of an object in one cell into the surrounding cells.

Advantage of cellular methods is their ability to combine spread with the conditions in the neighbouring cells, sometimes called **habitat**.

The habitat is provided externally, it can be the energy potential in a solid, the geographic and climatic conditions for the spread of plants and animals or the conditions of exchange and competition with other people.

Probability of dispersion

(population of cell by agent, e.g. new plant, new culture, new language, new word,…)

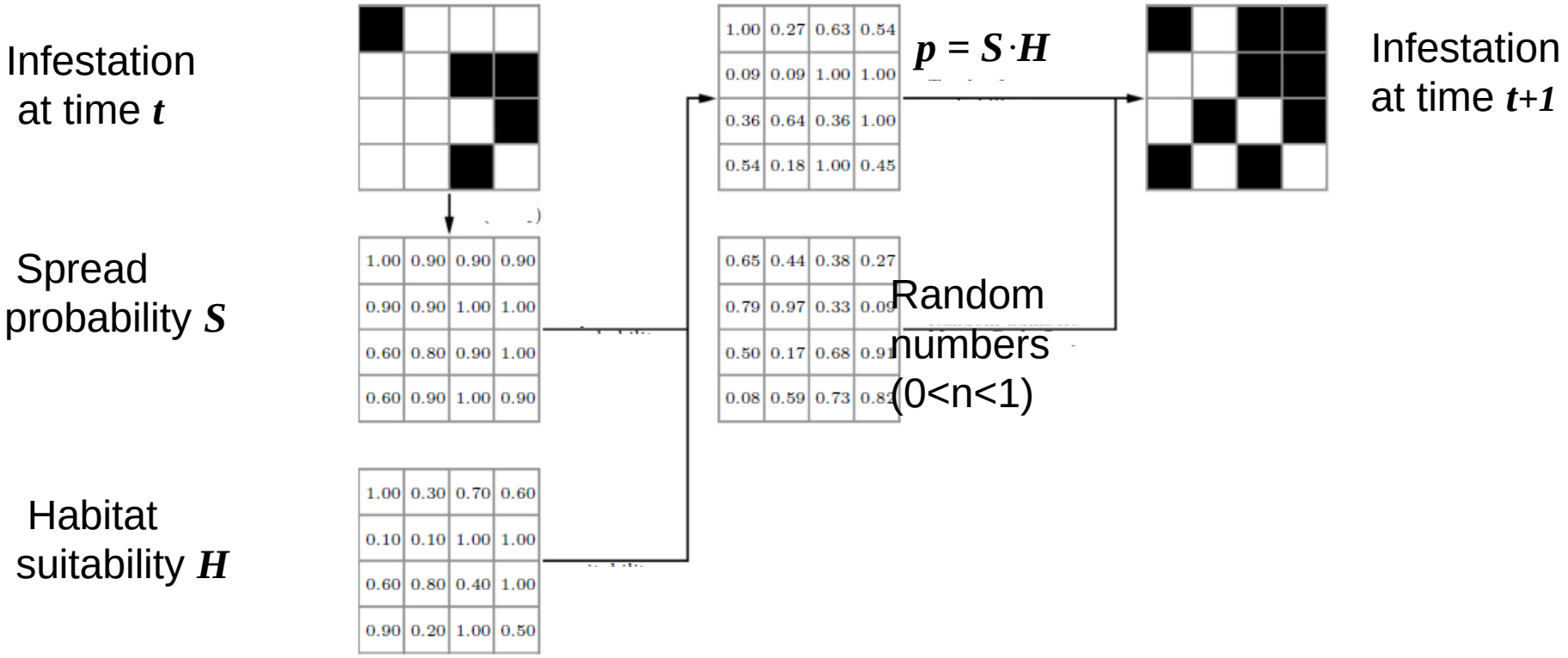
$$p = S \cdot H$$

S spread probability, Gaussian or long tail dispersion or other dispersion.
(dispersion not necessarily Gaussian)

H habitat function

Simplest case: yes / no (black/white)

The squares (“grid cells”) symbolize subdivision of space.

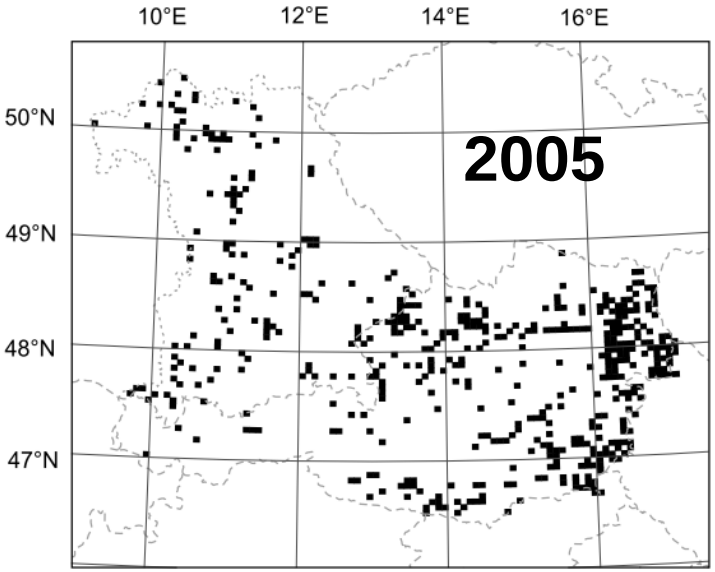


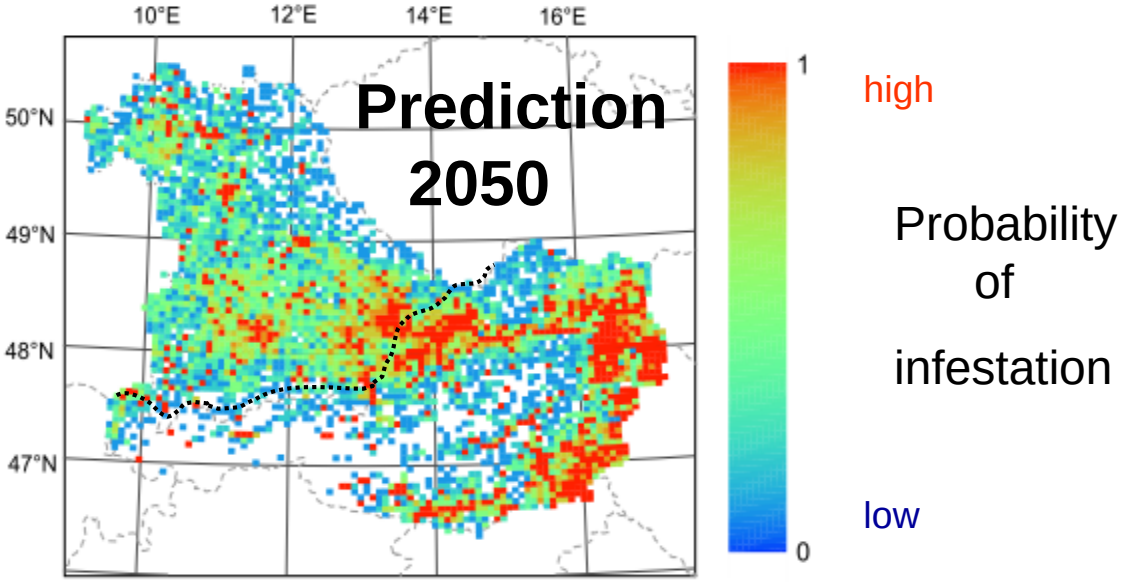
Neobiota

Habitat: climate, geographic conditions

Michael Leitner *Dispersal in Plants and Animals: Modelling*

Grid cells infested by ragweed





Poster 42: Robert Richter, Michael Leitner, G.V.
Diffusional spread and pollen load

Archaeology

Detlef Gronenborn and Carsten Lemmen

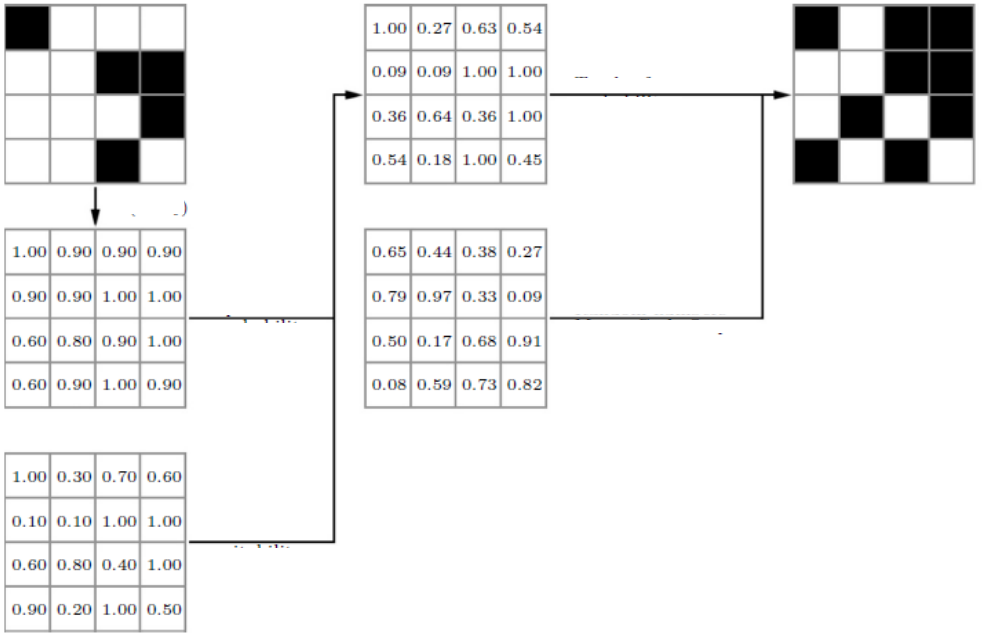
The Expansion of Farming as Seen from Archaeology and Related Disciplines

Cellular automaton for simulating the expansion of farming
(the spread of the Neolithic)

Again **Language**

Habitat: social conditions, „prestige“ of a language (Abrams and Strogatz 2003), schools, support/impediment from state‘s part

Simplest case: yes / no (black/white)
is no longer appropriate

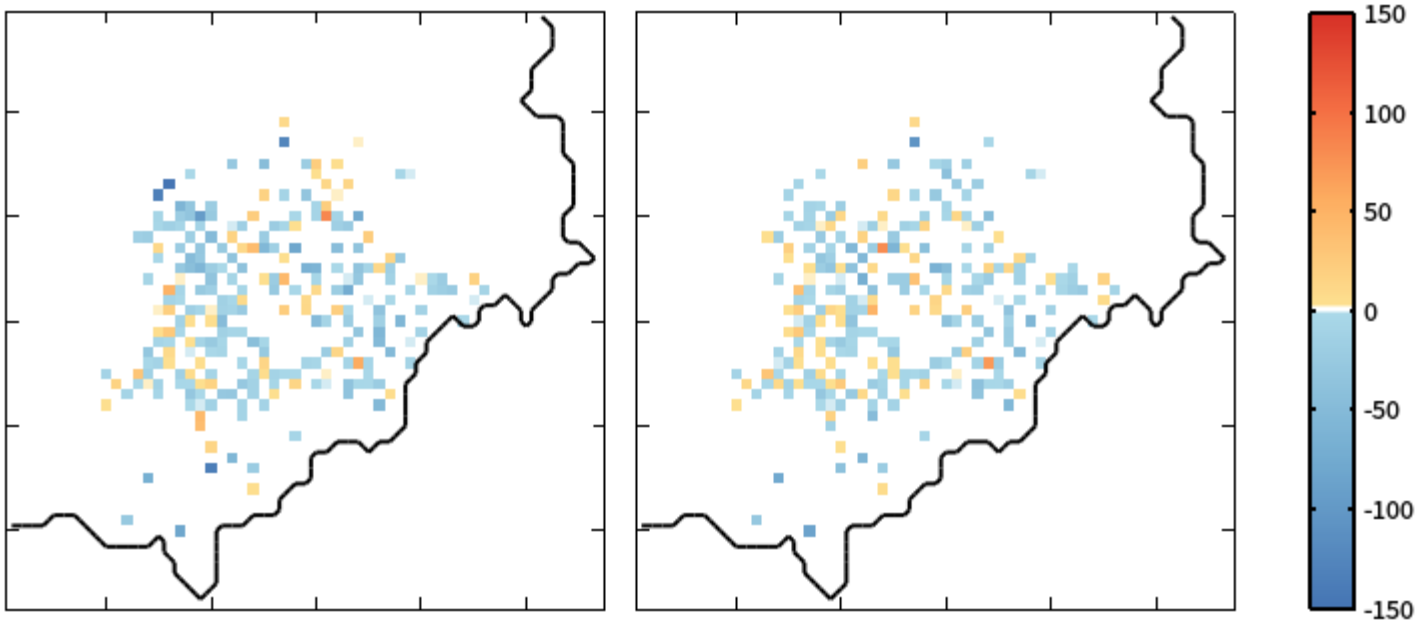


Need more gradation: number of speakers, “prestige”, etc.

Poster 40: Katharina Prochazka, G.V.
Modelling language shift in Carinthia, Austria

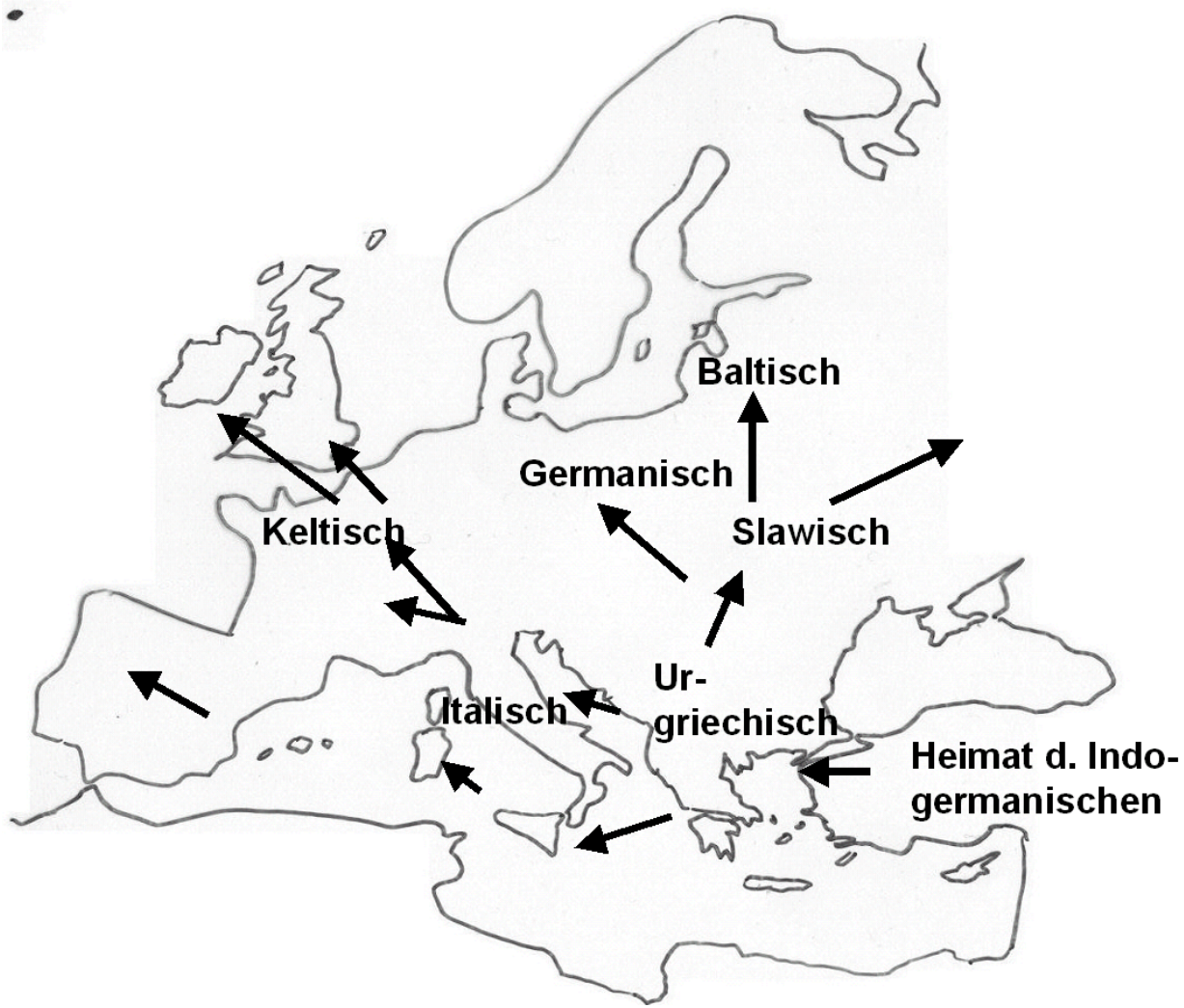
Data from *Statistics Austria*

Simulation



Change in number of speakers of Slovenian language from 1880 to 1900 in Southeastern Carinthia, Austria

In 20th century progressive retreat of minority language.
Understand reasons→discuss measures to save minority languages.



Spread of Indoeuropean following Renfrew (1982) from an Anatolian homeland

Russell Gray *Expansion of Language Families*

The danger and the reproach of reductionism are pending.
But please consider that for science it should be allowed,
if not even an imperative,
to transgress the limits of one's narrow discipline.

Understand the past:

Languages

Modern people

Prognostics:

Epidemies

Neophyta

Loss of languages

….

Thank you !